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For $AJ = AK + KJ = \frac{1}{2}(AC + AG) + KF = \frac{1}{2}b + \frac{1}{2}c + \frac{1}{2}a = s$, $AL = AJ - JL = s - a$, $CL = GJ = AJ - AG = s - c$, $CJ = AJ - AC = s - b$.

This solution is based on Legendre.

Also solved by J. L. Riley.

435. Proposed by R. M. MATHEWS, Riverside, Calif.

From a fixed point A perpendiculars are dropped to the tangents drawn to a circle whose center is O. Prove that the locus of the feet of the perpendiculars is a limaçon.

Solution by Walter C. Eells, U. S. Naval Academy.

Let a be the radius of the circle, and let distance OA be c. Take x-axis through O and A and y-axis through $O \perp OA$. Then the equation of the circle is $x^2 + y^2 = a^2$, and that of all tangents to it with the slope m is

$$y = mx \pm a\sqrt{1 + m^2}. \tag{1}$$

The equation of all lines through $A \perp$ to the system of lines given by (1) is

$$y = -\frac{1}{m}(x - c). \tag{2}$$

Transforming to new origin at A by equations $x = x' + c$, $y = y'$ and dropping primes, equations (1) and (2) become

$$y = m(x + c) \pm a\sqrt{1 + m^2}, \tag{3}$$

$$m = -\frac{1}{y}x. \tag{4}$$

Eliminating the parameter m by substituting (4) in (3), clearing of fractions, transposing and squaring, we have the well-known equation

$$(x^2 + y^2 + cx)^2 = a^2(x^2 + y^2),$$

representing the three standard forms of the limaçon, according as $c <, =, >, a$ i. e., according as A is within, on, or without the circle.

Also solved by J. W. Clawson, S. W. Reaves, Elijah Swift, A. M. Harding, Daniel Kreth, C. N. Schmall, Horace Olson, and the Proposer.

CALCULUS.

Additional solutions were received too late for the June issue from J. L. Riley for 335, and from A. M. Harding and J. W. Clawson for 345.

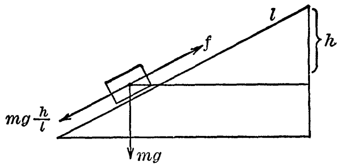
346. Proposed by C. N. SCHMALL, New York City.

Give the height of an inclined plane, to determine its length so that a given force acting on a given mass parallel to the plane may draw it up in the shortest time.

Solution by B. L. Libby, Ann Arbor, Mich.

Using the notations indicated in the figure, we have the resultant force acting upwards

$$f - mg \cdot \frac{h}{l}.$$



The acceleration required is therefore given by

$$a = \frac{fl - mgh}{ml}$$

Hence $dl/dt = (fl - mgh/ml)t$, and $l = (fl - mgh/2ml)t^2$, from which

$$t^2 = 2ml^2/(fl - mgh).$$

Differentiating, we have

$$t\frac{dt}{dl} = \frac{mfl^2 - 2m^2ghl}{(fl - mgh)^2}.$$

The usual test for a minimum for t gives

$$l = \frac{2mgh}{f} \quad \text{and} \quad t = 2m\sqrt{\frac{2gh}{f}}.$$

Also solved by Horace Olson and P. Peñalver.

347. Proposed by R. D. CARMICHAEL, Indiana Univ.

Show that the differential equation

$$9\left(\frac{d^2y}{dx^2}\right)^2\frac{d^5y}{dx^5} - 45\frac{d^2y}{dx^2}\frac{d^3y}{dx^3}\frac{d^4y}{dx^4} + 40\left(\frac{d^3y}{dx^3}\right)^3 = 0$$

remains unchanged when the variables x and y undergo any projective transformation.

Solution by A. M. Harding, University of Ark.

Any projective transformation can be reduced to a succession of integral transformations of the form

$$x = aX + bY + c, \qquad y = a_1X + b_1Y + c_1, \tag{1}$$

combined with the particular transformation

$$x = \frac{1}{X}, \qquad y = \frac{Y}{X}.$$

Hence it will be sufficient to show that the given equation remains invariant under each of these transformations. From (1) we find

$$y' = \frac{a_1 + b_1Y'}{a + bY'}, \; y'' = \frac{(ab_1 - a_1b)Y''}{(a + bY')^3}, \; y''' = \frac{(ab_1 - a_1b)[(a + bY')Y''' - 3b(Y'')^2]}{(a + bY')^5},$$